A Study on Involute Helical Gears. III : In the Case of Pinion without Dedendum and Gear without Addendum

A Study on Involute Helical Gears. III
In the Case of Pinion without Dedendum and Gear without Addendum*

Masao NARUSE** and Saburo TAKANASHI

The Research Institute for Scientific Measurements

(Received November 27, 1952)

Synopsis

In this paper, we report the studies on the mating of gear teeth in the case of the pinion without dedendum and the gear without addendum.

If the pinion is a driver, the gear teeth mates in arc of recess only, consequently, increases the smoothness of operation.

Accordingly, the gear of this type can be applied to a reduction gear. The equations for this tooth profile are expressed by equations (3-6), (3-7) and (3-8), and the equations for tooth thickness are obtained from the following equations given in the 1st. report:

$$t_1 = \frac{1+x''_{1h}\dfrac{2\pi}{z''_{1h}}}{\dfrac{\pi}{z''_{1h}}\left(1+x''_{1h}\dfrac{2\pi}{z''_{1h}}-\omega_1\right)}\left[\operatorname{inv}\left\{\cos^{-1}\left(\frac{\cos\alpha}{1+x''_{1h}\dfrac{2\pi}{z''_{1h}}}\right)\right\}-\operatorname{inv}\alpha\right], \tag{1-33}$$

$$t_2 = 1 - t_1 \,. \tag{1-34}$$

The gear of this type can be designed by using these equations.

In this report, in contrary to the case in the 2nd. report, we considered the case of pinion without dedendum and gear without addendum as shown in Fig. 1. In this case, when we drive the pinion to speed up and gear to couple with it, the contacts of gears bigin at point P and ends at G as shown in the figure, and they are shown as the contacts along the arc of recess only. Accordingly, when we apply the gear of this type to a pair of the reduction gear, the sliding direction become constant for the same reason discussed in the 2nd. report, and it is considered about the general equation of the tooth profile of this type. Also in this case, as in the 2nd. report, from the general

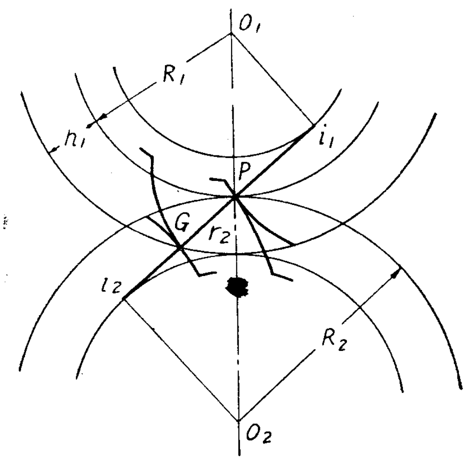

Fig. 1

* Published in a Lecture Meeting of Institute of the Mechanical Engineering of Japan in Tohoku District, Dec., 1949.

** Precision Machinery Department.

equations of helical gears (1–25), (1–26) and (1–27) descrived in the 1st. report, can be introduced easily.

As

$$\sum{}_2' = \frac{r_2(1+\varepsilon)}{\varepsilon R_1 \sin\alpha - r_2} = \text{constant}\,, \tag{3-1}$$

let

$$c = 0\,, \tag{3-2}$$

$$\sum{}_1' = c\sum{}_2' = 0 \tag{3-3}$$

that is,

$$\sum{}_1' = \frac{r_1\left(1+\frac{1}{\varepsilon}\right)}{R_1 \sin\alpha - r_1} = 0\,,$$

then

$$r_1 = 0\,. \tag{3-4}$$

But from (1–9)[1],

$$r_1 = -R_2 \sin\alpha + \sqrt{h_2((2R_2+h_2)+R_2^2\sin^2\alpha}\,.$$

Then, from this equation and (3–4),

$$h_2 = 0\,. \tag{3-5}$$

This is shown by the figure which corresponds to Fig. 1 which is in the case of the pinion without dedendum and the gear without addendum.

Therefore, when we substitute the above values in (1–25), (1–26) and (1–27), we have

$$z''_{1h} = (n - B\tan\beta\cos\alpha)\frac{2\pi}{K\tan\alpha}\,\frac{\varepsilon+\sum_2'+1}{\varepsilon\sum_2'}\,, \tag{3-6}$$

$$x''_{1h} = (n - B\tan\beta\cos\alpha)\frac{1}{K\tan\alpha}\,\frac{\varepsilon+\sum_2'+1}{\varepsilon\sum_2'}\times$$

$$\left[-1+\sqrt{1+\left\{\left(1+\frac{\varepsilon\sum_2'}{1+\varepsilon+\sum_2'}\right)^2-1\right\}\sin^2\alpha}\,\right] \tag{3-7}$$

$$x''_{2h} = 0\,. \tag{3-9}$$

The three equations z''_{1h}, x''_{1h} and x''_{2h} come to a common equations for the tooth profile of this type.

Next, when we compare the equation (3–6) for the number of teeth in this case with the equation (1–25) for the general case,

$$z''_{1h} = (n - B\tan\beta\cos\alpha)\frac{2\pi}{K\tan\alpha}\,\frac{\varepsilon+\sum_2'+1}{\varepsilon\sum_2'}\,,$$

$$z_{1h} = (n - B\tan\beta\cos\alpha)\frac{2\pi}{K\tan\alpha}\,\frac{c\varepsilon-(c\varepsilon-1)K}{\varepsilon(c+1)-(c\varepsilon-1)K}$$

Now, if

$$\sum{}_2' = \sum\,,$$

$$z_{1h} = \frac{1+\varepsilon+\sum}{(1+\varepsilon)(\sum+2)}\,z''_{1h}$$

and if

$$\frac{1+\varepsilon+\sum}{(1+\varepsilon)(\sum+2)} = b$$

$$b < 1\,,$$

(1) Sci. Rep. RITU., **A1** (1949), No. 4

therefore, $z''_{1h} > z_{1h}$.

Accordingly, the tooth profile of this type is recognized as that which supplies the loss of the arc of contact by increassing the number of teeth. When the number of teeth in contact in section perpendicular to the axis n_s is 1, we have

$$n - B \tan\beta \cos\alpha = n_s = 1 .$$

In this case, if we show it by a diagram, we will get curvetures as shown in Fig. 2 AB, and as mentioned before, three equations of z''_{1h}, x''_{1h} and x''_{2h} determines the outline of the half side of the tooth profile.

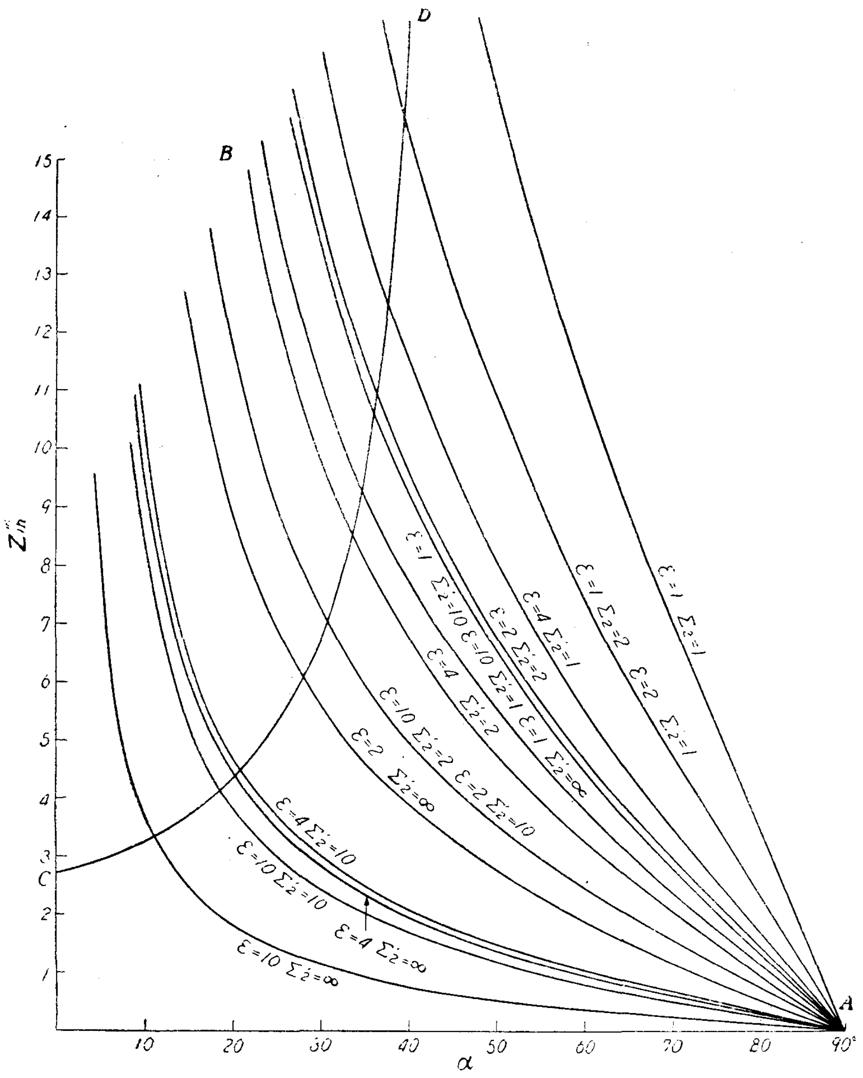

Fig. 2

In this case, it is common that the drivings of gears are posible in the regular or reversible motion of the same. And, at this point, we must consider the tooth thickness. We may use the equation of the relation (1-33), (1-34) and (1-35), (1-36) described in the 1st. report. But in this case,

$$x''_{2h} = 0$$

and $$T_{a2} = T_{p2} , \qquad \therefore \quad \omega_2 = 1 .$$

Therefore, we use the above equations for the tooth thickness, (1-33), (1-34), and it may be good if we calculate by substituting the value of z''_{1h} and x''_{1h} in this equation.

Next, we will consider the minimum number of teeth as the reversible gear. As described already,

$$\omega_2 = 1 .$$

But in this case, by the calculation, the number of teeth will be the minimum in which values t_2 and ω_1 will be

$$t_2 = 0 ,$$
$$\omega_1 = 0 .$$

In the above case, it may be recognized that the number of teeth will be the minimum. In other word, we understand that when the ends of gear and pinion are sharp; it gives the minimum number of teeth. Let us substitute the above mentioned values in the relation of tooth thickness

$$\operatorname{inv}\left\{\cos^{-1}\left(\frac{\cos\alpha}{\sqrt{1+\left\{\left(1+\frac{2\pi}{z''_{1h}}\frac{1}{\tan\alpha}\right)^2-1\right\}\sin^2\alpha}}\right)\right\}-\operatorname{inv}\alpha = \frac{\pi}{z''_{1h}} . \qquad (3\text{-}11)$$

There, similarly, as in the above calculation,

$$n - B\tan\beta\cos\alpha = n_s = 1 .$$

By this, it means that if we let the fugure leting α be coordinate and z''_{1h} be absissa, the group of curveture as shown in Fig. 2 *CD* will be made. The minimum number of teeth as reversible gear is satisfied by the above mentioned equations, z''_{1h}, x''_{1h}, x''_{2h}, and the tooth thickness. Therefore, it can be made from two groups of carvetures, *AB* and *CD*, as in Fig. 2. In Fig. 2, for example, in the case $\varepsilon = 1$, we have

$$\begin{aligned} \textstyle\sum_2' &= 1 & z''_{1h} &= 22 \\ \textstyle\sum_2' &= 2 & z''_{1h} &= 16 \\ \textstyle\sum_2' &= 10 & z''_{1h} &= 11 \\ \textstyle\sum_2' &= \infty & z''_{1h} &= 10 \end{aligned}$$

This is the case of the number of teeth in contact in section perpendicular to the axis, $n_s=1$. If the helix angle is $\beta=0$, it becomes the case of spur gear, and it is considered to be the case in which the number of teeth in contact of spur gear is 1. Therefore, when we use this type to the spur gear, the minimum number of teeth become too much compared with the common case. Accordingly, the gear of this type is said to be that of "A-Verzerung", when we use it in the practical case, a disgn will be difficult in the points of the number of teeth and number of teeth in contact of gears.

On the contrary, in the case of the involute helical gears, the number of teeth in contact in section perpendicular to the axis, n_s, can be smaller than 1, and decrease in this, n_s, makes up for the increase in the value of $B{\cdot}\tan\beta{\cdot}\cos\alpha$, and the number of teeth in contact of the whole helical gears n will be as follows:

$$n = n_s + B.\tan\beta\cos\alpha \geqq 1.$$

And then, it may be perfect contact. In the case in which the number of teeth in contact in section perpendicular to the axis, n_s is smaller than 1, we determined how the minimum number of teeth will be changed.

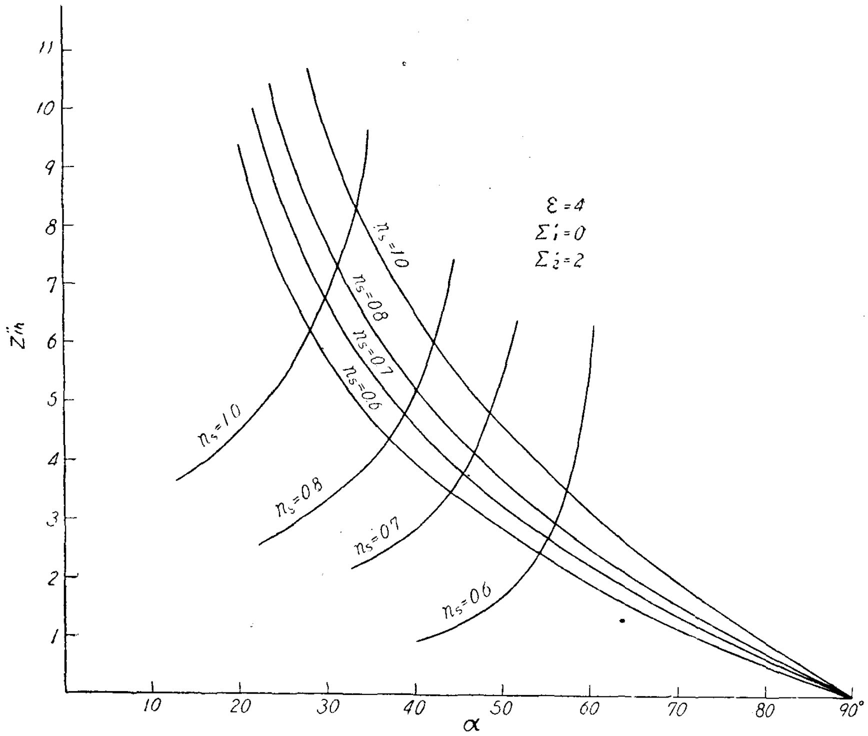

Fig. 3

For example, $\varepsilon = 4, \ \sum_1{}' = 0, \ \sum_2{}' = 2,$

$n_s = 1.0, \ 0.8, \ 0.7, \ 0.6.$

Then $n_s = 1.0, \quad z_{1h}'' = 9,$

$n_s = 0.8, \quad z_{.h}'' = 6,$

$n_s = 0.7, \quad z_{1h}'' = 4,$

$n_s = 0.6, \quad z_{1h}'' = 3.$

As mentioned above, we can make the minimum number of teeth smaller, and therefore, the tooth profile larger, and make it endure under the heavy load. And if the number of teeth in contact in section perpendicular to the axis, n_s may be small, by making up for values of $B\cdot\tan\beta\cdot\cos\alpha$, it can be taken to be large number of teeth in contact of involute helical gear, and we can expect the smooth driving.

Next, for an example of the above calculations, by using each equations described already, we designed the reduction gears that make the number of revolution of the gas turbine engine 75000 r.p.m down to 1500 r.p.m in one step.

For the material of pinion, we used Ni-Cr steel, and assumed that the diameter of the pinion shaft to be 10 mm for the shaft horse power 75 HP.

There is a case in which pinion is addendum only, and we took 10.5 mm for the diameter of the pitch curcle. And as the speed reduction ratis is $\varepsilon = 50$, the diameter of pitch circle of gears is 52.5 cm.

There, we designed for three cases as follows: They are cases which the number of teeth of pinion is 1, 2 and 3, and the number of teeth of gears is 50, 100 and 150. And in these cases, we decided as the specific sliding are each

$\sum_1' = 0, \qquad \sum_2' = 2,$

and the halix angle are $\beta = 30°$.

The 1st. plan: In the case of the number of teeth, 3 and 150,

from the equation z_{1h}'' (3-6), $n_s = 0.644$,

from the equation x_{1h}'' (3-7), $x_{1h}'' = 0.420$,

from the equation of the tooth thickness,

$t_1 = 0.744$, as $\omega_1 = 0$,

circular pitch $p = 10.966$ mm

If we show the outline of the tooth profile in section perpendicular to the axis as shown in Fig. 4, it will be as follows:

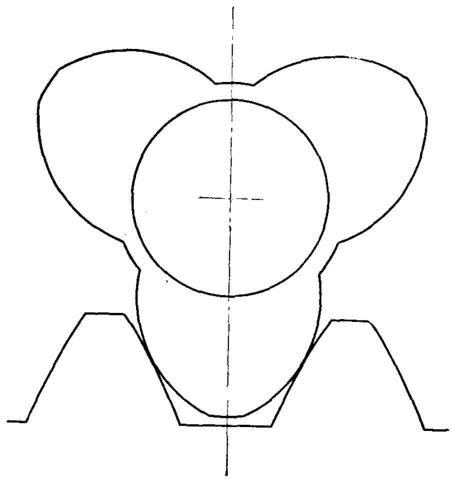

Fig. 4

The 2nd. plan: In the case of the number of teeth, 2 and 100;

from the equation

z_{1h}'' (3-6), $n_s = 0.624$,

from the equation

x_{1h}'' (3-7), $x_{1h}'' = 0.307$

from the equation of the tooth thickness, $t_1 = 0.910$, as $\omega_1 = 0$, circular pitch $p = 16{,}493$ mm.

If we show the outline of the tooth profile in section perpendicular to the axis as shown Fig. 5, it will be as follows:

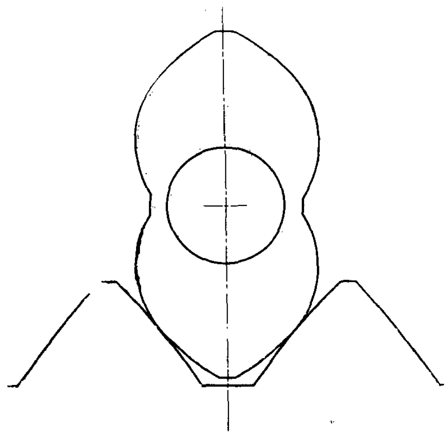

Fig. 5

The 3rd. plan: In the case of the number of teeth, 1 and 50,

from the equation

z_{1h}'' (3-6), $n_s = 0.531$,

from the equation

x_{1h}'' (3-7), $x_{1h}'' = 0.285$,

from the equation of the tooth thickness, $t_1 = 0.936$, as $\omega_1 = 0$, circular pitch $p = 32.987$ mm.

If we show the outline of the tooth profile in section perpendicular to the oxis as shown Fig. 6, it will be as follows:

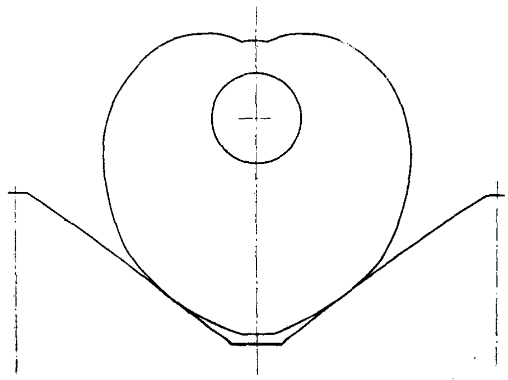

Fig. 6

Further, in the 1st., 2nd. and 3rd. plan, when we calculate the breadth of gear from the points of the strength, as double-helical gear are obtained as shown in Fig. 7 (a), (b) and (c). Accordingly, we determined the breadth of gear as mentioned above, and also calculated the number of teeth in contact of the whole involute helical gears.

We have, in the 1st. plan, $n = 5.474$,
in the 2nd. plan, $n = 3.495$
in the 3rd plan, $n = 2.316$

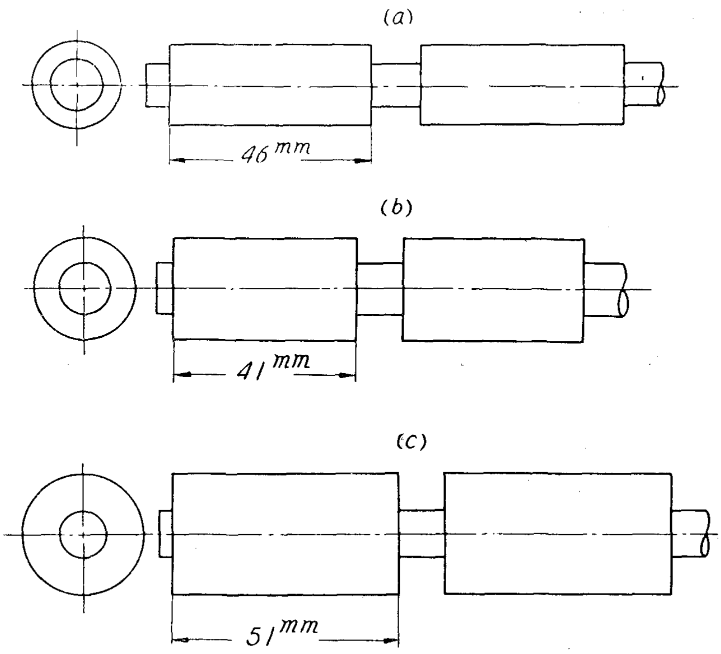

Fig. 7

In addition, the model of these typed gears are shown in photos 1, 2 and 3.

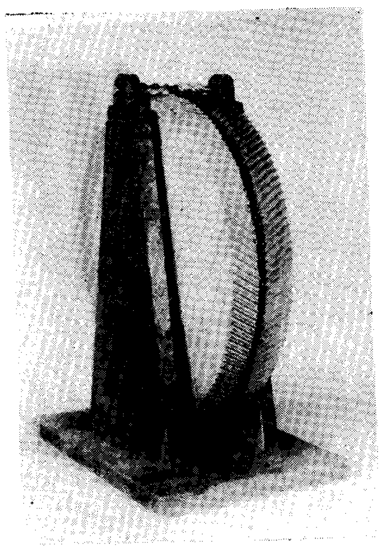

Photo. 1

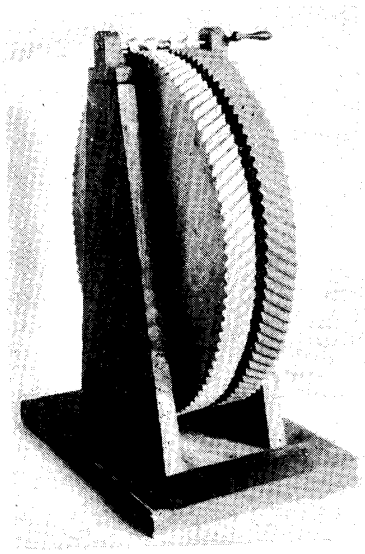

Photo. 2

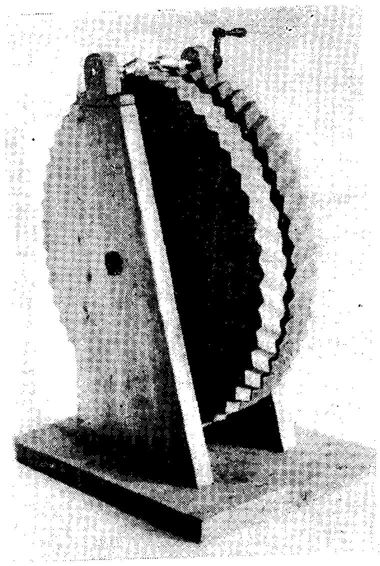

Photo. 3